Rectangles

In early 19th century, the ruler Hoseyngulu Khan Sardar orders to build a palace on a plateau overseeing Zangi river. The plateau is modeled as an $n \times m$ grid of square cells. The rows of the grid are numbered 0 through $n-1$, and the columns are numbered 0 through $m-1$. We refer to the cell in row i and column j ($0 \leq i \leq n-1, 0 \leq j \leq m-1$) as cell (i, j). Each cell (i, j) has a specific height, denoted by $a[i][j]$.

Hoseyngulu Khan Sardar asked his architects to choose a rectangular **area** to build the palace. The area should not contain any cell from the grid boundaries (row 0, row $n-1$, column 0, and column $m-1$). Hence, the architects should choose four integers r_1, r_2, c_1, and c_2 ($1 \leq r_1 \leq r_2 \leq n-2$ and $1 \leq c_1 \leq c_2 \leq m-2$), which define an area consisting of all cells (i, j) such that $r_1 \leq i \leq r_2$ and $c_1 \leq j \leq c_2$.

In addition, an area is considered **valid**, if and only if for every cell (i, j) in the area, the following condition holds:

- Consider the two cells adjacent to the area in row i (cell $(i, c_1 - 1)$ and cell $(i, c_2 + 1)$) and the two cells adjacent to the area in column j (cell $(r_1 - 1, j)$ and cell $(r_2 + 1, j)$). The height of cell (i, j) should be strictly smaller than the heights of all these four cells.

Your task is to help the architects find the number of valid areas for the palace (i.e., the number of choices of r_1, r_2, c_1 and c_2 that define a valid area).

Implementation details

You should implement the following procedure:

```
int64 count_rectangles(int[][] a)
```

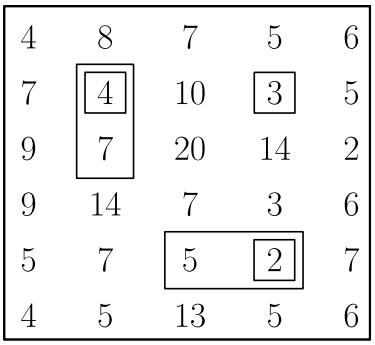
- a: a two-dimensional n by m array of integers representing the heights of the cells.
- This procedure should return the number of valid areas for the palace.

Examples

Example 1

Consider the following call.

```
count_rectangles([[4,  8,  7,  5,  6],
                  [7,  4,  10, 3,  5],
                  [9,  7,  20, 14, 2],
                  [9,  14, 7,  3,  6],
                  [5,  7,  5,  2,  7],
                  [4,  5,  13, 5,  6]])
```



:There are 5 valid areas, listed below

- $r_1 = r_2 = c_1 = c_2 = 1$
- $r_1 = 1, r_2 = 2, c_1 = c_2 = 1$
- $r_1 = r_2 = 1, c_1 = c_2 = 3$
- $r_1 = r_2 = 4, c_1 = 2, c_2 = 3$
- $r_1 = r_2 = 4, c_1 = c_2 = 3$

For example $r_1 = 1, r_2 = 2, c_1 = c_2 = 1$ is a valid area because both following conditions :hold

- is strictly smaller than $a[0][1] = 8$, $a[3][1] = 14$, $a[1][0] = 7$, and $a[1][1] = 4$.$a[1][2] = 10$
- is strictly smaller than $a[0][1] = 8$, $a[3][1] = 14$, $a[2][0] = 9$, and $a[2][1] = 7$.$a[2][2] = 20$

Constraints

- $1 \leq n, m \leq 2500$
- (for all $0 \leq i \leq n-1, 0 \leq j \leq m-1$) $0 \leq a[i][j] \leq 7\,000\,000$

Subtasks

1. $n, m \leq 30$ (points 8)
2. $n, m \leq 80$ (points 7)
3. $n, m \leq 200$ (points 12)
4. $n, m \leq 700$ (points 22)

5. $n \leq 3$ (points 10)
6. $0 \leq a[i][j] \leq 1$ (for all $0 \leq i \leq n-1, 0 \leq j \leq m-1$ points) 13)
7. No additional constraints (28 points).

Sample grader

The sample grader reads the input in the following format:

- line 1: $n\ m$
- line $2+i$ (for $0 \leq i \leq n-1$): $a[i][0]\ \ a[i][1]\ \ \ldots\ \ a[i][m-1]$

The sample grader prints a single line containing the return value of `count_rectangles`.